Control-aware Uplink Resource Allocation for Cyber-Physical Systems in Wireless Networks

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Abstract—Cellular networks beyond LTE must address the requirements of Machine Type Communications (MTC). Many MTC applications fall into category of the cyber-physical systems, i.e., feedback systems with the physical processes integrated with networks and computation. The challenge of CPS is to design the communication network to support the underlying feedback control loop. To address this challenge, we adopt a cross-layer approach to scheduling in wireless networks. We formulate the resource allocation problem with the objective of maximizing the control performance in terms of the network-induced error. Following that, we devise a Maximum Predicted Error First (MPEF) scheduler, which provides a close to optimal performance while only relying on offline information about the control loops. In a case study of LTE cell, we compare MPEF with state-of-the-art scheduling algorithms.

I. INTRODUCTION

Upcoming evolution of cellular networks from 4G to 5G is largely driven by changing application demands. Instead of solely focusing on data rate increase, 5G is envisioned to be capable of satisfying a broad range of requirements for Machine Type Communication (MTC), including vehicular applications, industrial automation and cyber-physical systems. In particular, METIS 2020 project defines two important MTC use cases: ultra-reliable and ultra-low latency Machine Type Communication (uMTC) and massive Machine Type Communication (mMTC) [1], [2]. In the recent years, significant amount of research contributions has been done on re-designing 3GPP Long Term Evolution (LTE) in order to support both uMTC and mMTC [3], [4], [5]. In particular, understanding distinct features of MTC in comparison to conventional Human-to-Human (H2H) applications can lead to efficient designs of communication solutions tailored to MTC applications.

A common challenge of many MTC applications, especially cyber-physical systems, is the fact that the underlying application is a feedback control loop, whereas the communication end points are the sensor, measuring the controlled plant’s output, and the respective remote controller, reacting to the sensor’s data. Many conventional MTC applications, such as smart grids or industrial automation rely on a control loops as underlying set-up. Clearly, the requirements of control based applications are inherently different from H2H, and their peculiarities are often ignored in the current wireless systems. To address this challenge, we adopt a cross-layer design approach: by considering detailed models of feedback control loops, we replace the traditional quality of service with quality of control, and use it as an efficiency metric for resource allocation in wireless radio access networks.

A. Related Work

Scheduling and medium access control (MAC) for control applications – often referred to as networked control systems (NCS) – have been previously studied in the control community. Effects of delay and packet drops on the stability and performance of feedback control loops are discussed in [6], [7]. A lot of works have been targeting optimization techniques for decentralized MAC (CSMA/CA, ALOHA-based protocols) [8], [9], [10], with special attention to WLAN for NCS [11], [12], [13]. There is however less related work on the centralized MAC problems. The problem of joint control and rate allocation is considered in [14], [15]. Centralized approach for resource allocation, formulated as scheduling problems, has been developed in [16], but only for simple network models.

A well-known Maximum Error First Try-once-Discard approach for scheduling of feedback control systems has been presented in [17], [18]. The authors propose to schedule the control sub-system with the highest control error first, in a greedy fashion. However, this approach requires the scheduler to know the error of each control sub-system prior to making a scheduling decision. Obtaining this knowledge requires additional overhead, and, for some scenarios is simply not possible, since it requires an estimator at the sensor side.

Fig. 1: Scenario: N control loops are coupled via a wireless communication network, with resource allocation running at the base station.
in the uplink case [19]. In contrast to state-of-the-art, we approach the control-aware communication problem from the performance, and not from the stability point of view.

B. Contributions

In this paper, we study a scenario where multiple control systems are contending for uplink wireless resources in a cellular radio access network (RAN). We address the limitations of the state-of-the-art by (i) adapting existing control-aware scheduling problem to the wireless resource allocation problem for cellular networks. (ii) Next, we devise a scheduling algorithm for allocating uplink wireless resources, which does not require knowledge of the instantaneous control error at the base station, and operates with prediction of network-induced error based on the offline control system parameters. The algorithm remains both control-aware and channel-aware. (iii) We present a case study with a single LTE cell and inverted pendulum as exemplary control system, evaluating the performance of different scheduling algorithms.

The remainder of the paper is organized as follows. In Sec. II we introduce the considered scenario and define the resource allocation problem. Next, in Sec. III we define quality of control metrics and control-aware scheduling policies. Sec. IV presents the results of the case study; finally, we conclude with Sec. V.

II. SCENARIO AND PROBLEM FORMULATION

We consider a networked control system of $N$ independent linear time invariant (LTI) control sub-systems, each consisting of a plant $P$, controller $C$, and sensor $S$ (see Fig. 1). The behavior of the plant process of the $i$th sub-system is described by a difference equation:

$$x_{k+1} = A_i x_k + B_i u_k + w_k,$$

where $x_k$ is the state vector, $u_k$ is the control input, and $w_k$ is the noise at time-step $k$. The matrices $A_i, B_i$ describe the system and input matrices. The noise $w_k$ is considered to be independent and identically distributed (i.i.d) vector, distributed according to a zero-mean multivariate normal distribution with the covariance matrix $W_i$.

Plant and controller are assumed to be co-located, however, the sensor is measuring output of the plant remotely and, hence, the measurements have to be sent via a wireless network. We consider a scenario where all the sensors send an uplink transmission towards a base station, from where the packet is delivered via the backhaul network to the respective controllers and plants. In the following, we assume an uncongested backhaul and, therefore, ignore its impact on the control systems. We further assume that the sensors are synchronized and have equal sampling periods.

A. Wireless Resource Allocation

The resource allocation for sensor transmissions is done by a central scheduler located at the base station (BS). We assume an exemplary wireless system based on the LTE RAN, see Fig. 2. While taking a decision, the base station has information about the current buffer size and uplink channel quality of a UE via Buffer Status Reports (BSR) and Sounding Reference Signals (SRS), respectively. The base station allocates the necessary amount of resource blocks (RBs) to satisfy the demand of a given sensor. We assume that the scheduling decision is taken with the same periodicity as the sampling time of a control system. In our exemplary setup, this corresponds to scheduling once in a 10 ms long LTE frame. Correspondingly, we ignore short-term channel quality variations (i.e., fast fading effects). The smallest allocated resource unit, in other words the scheduling granularity, is a single RB, i.e., a $t_{RB} = 0.5$ ms long slot in time domain and $\Delta f = 180$ kHz bandwidth in frequency domain.

Scheduling the transmissions of control systems is fundamentally different from the conventional applications, e.g., video streaming, due to the fast cycle of “out-dating” of information. That is, if the $i$th control system’s state information is not transmitted during the sampling interval $k$, it becomes out-of-date as soon as the new state measurements are available. Hence, while scheduling the next transmission, only the new measurements should be sent, and the out-of-dated packet is to be discarded (Try-once-Discard [17]). The consequence is that the buffer of any sensor contains only a single packet of length $S$ bytes. We define the resource demand of the sensor $i$ in frame $k$ as the minimum number of RBs required to guarantee a successful transmission:

$$d_k = \left\lceil S / r_k \right\rceil \text{ RBs},$$

where $r_k$ defines the expected throughput per resource block, and depends on the uplink channel quality of sensor $i$. Hence, if we define the total number of available uplink RBs per frame as $R$, and the vectors of utilities and demands as $c_k, d_k \in \mathbb{Z}^N$, respectively, the scheduling problem can be formulated as:

$$\arg\max_{q_k} q_k^\top c_k \quad \text{s.t.} \quad q_k^\top d_k \leq R,$$
where \( q_k \) is the allocation vector with elements \(^1\)
\[
q_k^i = \begin{cases} 
1 & \text{if sensor } i \text{ is scheduled}, \\
0 & \text{if sensor } i \text{ is blocked}. 
\end{cases}
\]  

(4)

Due to the fact that the above defined scheduling problem is NP-hard, greedy heuristics are typically applied. However, we make two important simplifications in this work: (1) channel quality for given sensor is constant in the frequency domain, i.e., it is the same for all RBs; and (2) channel quality does not vary within frames. Both assumptions are typical and hold for low-mobility applications (e.g., Smart Grid) and relatively narrowband communication systems [20], [21].

These simplifications allow us to treat the scheduling problem defined by Eqn. (3) as a 0/1 knapsack problem, which can be solved in pseudo-polynomial time by dynamic programming algorithms [22]. In the following, we consider both optimal (solved by knapsack) algorithms, and greedy low complexity heuristics.

III. CONTROL AWARE RESOURCE ALLOCATION

A. Network-Induced Error

In order to choose the utility function properly, we need to define a quality of control (QoC) metric. In general, the plant behavior depends both on network and control law. To make the metric independent from the controller type, we assume that the control law is described by the mapping of past observations:

\[
\begin{align*}
    u^i_k &= -L_i E[x^i_k|Z^i_k]\,
    E[x^i_k|Z^i_k] &= (A_i-B_iL_i)E[x^i_{k-1}|Z^i_{k-1}] 
\end{align*}
\]  

(5)

where \( Z^i_k \) is the observation history of the \( i \)-controller, and \( L_i \) is the stabilizing feedback gain. A model-based estimator \( E[x^i_k|Z^i_k] \) is used if the sensor \( i \) is blocked from transmission.

Following that, we define the network-induced estimation error \( e^i_k \) as the difference between the estimated and actual states of the system:

\[
e^i_k = x^i_k - E[x^i_k|Z^i_k].
\]  

(6)

It has been shown in the previous work, that the network-induced error can be considered independent of the system state [16], [8], [9]:

\[
e^i_{k+1} = (1-q^i_k)A_i e^i_k + w^i_k.
\]  

(7)

Following this, we define a control performance metric as the expected quadratic norm of the network induced error:

\[
E[e^T_k e_k] = \sum_{i=1}^N E[e^T_k e^i_k] = \sum_{i=1}^N E[\|e^i_k\|^2].
\]  

(8)

\(^1\)We assume that Hybrid ARQ guarantees packet delivery up to the desired reliability level, hence, we do not consider packet loss, and transmission success is only determined by the allocation variable \( q^i_k \).

B. Control aware Utility Choice

1) Maximum Error First: As outlined in Subsec. I-A, several control-oriented scheduling metrics have been considered so far. Most prominent example is Maximum-Error-First (MEF) scheduling [17], [18]. In the case of MEF, the utility for resource allocation at a step \( k \) is:

\[
c^k_i = [\|e^i_k\|^2,\|e^2_k\|^2,\ldots,\|e^N_k\|^2]^T,
\]  

(9)

While the MEF approach is proven to be optimal for minimizing the network-induced error, it is challenging to implement it in real systems. That is, for the uplink transmission scenario, two prerequisites have to be met: first, for computing the error, a sensor must have an estimator to calculate the difference between the real and estimated state vector. With the sensors being typically of low complexity, this assumption is hardly realistic for most of the systems [19]. Secondly, collecting the error information from all sensors at the base station introduces high overhead and latencies.

2) Proposed Prediction based Control aware Scheduler: To overcome the limitations of the MEF scheduler, we introduce the Maximum Predicted Error First (MPEF) Scheduler, which relies solely on the knowledge of plant parameters: the system matrix \( A_i \) and the noise covariance matrix \( W_i \). Given these parameters, the quadratic network-induced error norm of user \( i \) at time-step \( k \) can be predicted. If we denote the time-step at which the subsystem \( i \) transmitted for the last time by \( t^i \), the expected value of the quadratic error norm \( E[e^T_k e^i_k] \) is obtained by:

\[
E[e^T_k e^i_k] = \sum_{p=0}^{k-t^i-1} tr((A^T_i)^p A^p_i W_i).
\]  

(10)

\( A^p \) represents the \( p \)-th power of any square matrix \( A_i \in \mathbb{R}^{n_i \times n_i} \) with \( A_i^0 \) being equal to the \( n_i \times n_i \) identity matrix.

Now, we use Eqn. (10) as a new utility function for the sensor \( i \) at time \( k \) for greedy allocation:

\[
c^i_k = \frac{E[e^T_k e^i_k]}{\max_{m} E[e^m e^T_k e^m_k]} r^i_k \quad \text{for } m \in [1, N],
\]  

(11)

and for optimal resource allocation:

\[
c^i_k = E[e^T_k e^i_k].
\]  

(12)

Greedy MPEF uses both the expected quadratic error norm and the expected data rate \( r^i_k \) while calculating the utility. Additionally, the expected value is normalized by the maximum value in the network and used for weighting the expected data rate.

In comparison to MEF, proposed MPEF approach does not require instantaneous knowledge of the network-induced error. The prediction only requires the knowledge of control system parameters \( A_i, W_i \), which do not vary over time for LTI systems. This makes it feasible to apply the policy for uplink direction as well as for downlink.
IV. Case Study

In this section, we evaluate the performance of the proposed MPEF against the state-of-the-art approaches, in a case study of $N$ independent discrete control systems, transmitting uplink sensor readings via a single-cell LTE network.

A. Control System Model

As an example of the open-loop unstable control system, we consider the inverted pendulum application: a pendulum mounted on a motorized cart [23] (see Fig. 3a). The controller’s objective is to hold the pendulum in an upright position by moving the cart back and forward. The state vector of the system is chosen as $[p_k, \dot{p}_k, \phi_k, \dot{\phi}_k]^T$, where $p$ and $\phi$ denote the cart position and pendulum angle, respectively. The equilibrium point is located at zero for both, i.e., $p = 0$ and $\phi = 0$. The matrices $A$ and $B$ for the discrete state-space representation with a sampling time of 10 ms are:

$$A = \begin{bmatrix} 1 & 0.01 & 0.0001 & 0 \\ 0 & 0.9982 & 0.0267 & 0.0001 \\ 0 & 0 & 1.0016 & 0.01 \\ 0 & -0.0045 & 0.3119 & 1.0016 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0001 \\ 0.0182 \\ 0.0002 \\ 0.0454 \end{bmatrix}$$

The Linear Quadratic Regulator (LQR) method is used to determine the stabilizing feedback gain $L$:

$$L = \begin{bmatrix} -61.9933 \\ -33.5040 \\ 95.0597 \\ 18.8300 \end{bmatrix}$$

To evaluate control-aware scheduling algorithms, we create diversity by using a second class of control systems, namely open-loop stable applications. Their system and input matrices are obtained from the inverted pendulum matrices as $\hat{A} = 0.8A$, and $\hat{B} = B$. The difference to the inverted pendulum is that this class of systems will converge to a certain finite state over time, while still occasionally requiring actuation to compensate for the performance degradation caused by the noise. In every simulation, half of the control systems are open-loop unstable, $A_i = \hat{A}$, $B_i = \hat{B}$, for $i \in [1, N/2]$; and half of them are open-loop stable, $A_i = \hat{A}$, $B_i = \hat{B}$, for $i \in [N/2 + 1, N]$.

Finally, the noise for all control systems is assumed to be zero-mean gaussian vector with covariance matrix:

$$W = \begin{bmatrix} 6.40 \cdot 10^{-9} & 0 & 0 & 0 \\ 0 & 4.90 \cdot 10^{-9} & 0 & 0 \\ 0 & 0 & 2.74 \cdot 10^{-5} & 0 \\ 0 & 0 & 0 & 4.87 \cdot 10^{-5} \end{bmatrix}$$

B. Wireless Network

We assume that a 1.4 MHz band is used with frequency division duplex (FDD) mode with symmetrical spectra for uplink and downlink (consistent with Cat. M1 introduced in 3GPP LTE Release 13 [24], [25]). Furthermore, we constructed a grid to simulate the cell model (see Fig. 3b). The BS is located at the middle point and sensors are uniformly distributed over a cell of dimension $500 \times 500$ m. The plants are located outdoors in an urban micro cell propagation scenario. The path loss and shadow fading are characterized by stochastic models defined by 3GPP [26]. The line-of-sight probability depends on the distance between sensor and BS [26].

The expected data rate is obtained from the Shannon’s capacity theorem. To make the simulations more realistic for LTE RAN, yet computationally simple, we use the approximation model introduced in [27], which adjusts the Shannon capacity to the LTE case by taking into account the bandwidth- and SNR efficiencies $\Delta f_{\text{eff}}$ and $\eta_{\text{eff}}$, and a correction factor $\eta$ of LTE (suitable values are taken from [27]). The resulting amount of data $r_k^i$ transmitted per resource block for sensor $i$ at time step $k$ is calculated as follows:

$$r_k^i = t_{\text{RB}} \times \Delta f \times \Delta f_{\text{eff}} \times \eta \times \log_2 \left(1 + \frac{\gamma_k^i}{\gamma_{\text{eff}}}\right) \text{Bytes}, \quad (13)$$

where $t_{\text{RB}}$ denotes RB duration, $\Delta f$ - RB bandwidth, $\gamma_k^i$ - Signal to Noise Ratio (SNR). Simulation parameters are listed in Table I.

C. Simulation Set-up

We benchmark the proposed MPEF scheduler to the state-of-the-art LTE scheduling policies, and to the state-of-the-art control-aware scheduling. In addition, for control-aware policies we consider the greedy resource allocation, and optimal, using dynamic programming solution to the resource allocation problem (formulated as 0/1 knapsack). This results in the following list of schedulers:

- Proposed MPEF, optimal (MPEF opt) and greedy (MPEF gre), with utilities defined by Eqns. (11) and (12), respectively.
(c) Inv. pendulum, state error

Value

(e) Inv. pendulum, avg. data rate

Network-induced error \( \bar{e} \)

(f) Inv. pendulum, violations

Number of requirement violations \( \sum_{i=1}^{N} \| e^i_k \| \).

Average data rate:

\[ \bar{R} = \frac{1}{T_D} \sum_{k=1}^{T_D} q_k d_k. \]  \hfill (14)

Average network induced error norm \( \bar{e} \):

\[ \bar{e} = \frac{1}{T_D N} \sum_{i=1}^{N} \sum_{k=1}^{T_D} \| e^i_k \|. \]  \hfill (15)

Average state error norm \( \bar{x} \):

\[ \bar{x} = \frac{1}{T_D N} \sum_{i=1}^{N} \sum_{k=1}^{T_D} \| x^i_k \|. \]  \hfill (16)

Number of nodes violating the maximum allowed pendulum deviation and cart position:

\[ |\phi_{\text{max}}| \leq 0.2 \text{ rad}, \]

\[ |p_{\text{max}}| \leq 0.4 \text{ m}. \]  \hfill (17)

- Proportional Fair (PF) [28]: resources are allocated proportionally, considering average data rate and instantaneous channel quality. Fairness factor \( \beta = 0.5 \).

Duration of every simulation run is \( T_D = 2000 \) sampling periods (corresponding to 20 s), and the simulation is performed with 30 runs for every number of sub-systems \( N \in [5, 15, 25, \ldots, 115] \) to generate 95 % confidence intervals. We record the following performance metrics:

- Average data rate:

\[ \bar{R} = \frac{1}{T_D} \sum_{k=1}^{T_D} q_k d_k. \]  \hfill (14)

- Average network induced error norm \( \bar{e} \):

\[ \bar{e} = \frac{1}{T_D N} \sum_{i=1}^{N} \sum_{k=1}^{T_D} \| e^i_k \|. \]  \hfill (15)

- Average state error norm \( \bar{x} \):

\[ \bar{x} = \frac{1}{T_D N} \sum_{i=1}^{N} \sum_{k=1}^{T_D} \| x^i_k \|. \]  \hfill (16)

- Maximum-Energy First, optimal (MEF opt) and greedy (MEF gre), as defined in III-B1, with utility as in Eqn. (9). We simulate it as a base line.

- Maximum throughput (MT) [28]: greedy scheduler, allocating the resources to the best channel quality UE first.

- Round Robin (RR) [28]: allocated to all UEs same amount of resources equally.

#### TABLE I: Simulation parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell dimensions</td>
<td>500 x 500 m</td>
</tr>
<tr>
<td>Minimum distance</td>
<td>to BS 11 m</td>
</tr>
<tr>
<td>Tx power (UE)</td>
<td>23 dBm</td>
</tr>
<tr>
<td>Center frequency</td>
<td>2 GHz</td>
</tr>
<tr>
<td>Antenna height</td>
<td>UE / BS 1.5 / 10 m</td>
</tr>
<tr>
<td>Shadowing: ( \sigma_{SP} )</td>
<td>Line-of-sight 3 dB</td>
</tr>
<tr>
<td></td>
<td>No line-of-sight 4 dB</td>
</tr>
<tr>
<td>Noise:</td>
<td>- figure 5 dB</td>
</tr>
<tr>
<td></td>
<td>- density -174 dBm/Hz</td>
</tr>
<tr>
<td>Efficiencies:</td>
<td>- bandwidth ( \Delta f_{\text{eff}} ) 0.83</td>
</tr>
<tr>
<td></td>
<td>- SNR ( \gamma_{\text{eff}} ) 1.25</td>
</tr>
<tr>
<td>Correction factor</td>
<td>( \eta ) 0.9</td>
</tr>
<tr>
<td>RB bandwidth ( \Delta f )</td>
<td>180 kHz</td>
</tr>
<tr>
<td>Packet size ( S )</td>
<td>750 Bytes</td>
</tr>
<tr>
<td>Channel coherence</td>
<td>time 180 ms</td>
</tr>
</tbody>
</table>

Fig. 5: Simulation results for the case study: network induced error \( \bar{e} \) with (a) and without (b) MT scheduler; avg. data rate \( \bar{R} \) per cell with (d) and without (e) MT scheduler; (c) state error \( \bar{x} \); (f) number of requirement violations (as defined in Eqns. (14)-(17), respectively). The simulations are performed over the number of nodes \( N \), with 0.95 confidence intervals.
An exemplary evolution of the pendulum states $p$ and $\phi$ as a function of time is depicted in Fig. 4. We can observe that during the 20 s run the position requirement is violated, i.e., $p$ leaves the allowed region $\pm 0.4$ m, while resource allocation was performed by RR scheduler.

**D. Simulation Results**

Figs. 5(a-f) accumulate resulting performance plots of the selected schedulers as we increase number of nodes $N$. Figs. 5a) and d) show the performance with maximum throughput scheduler. As expected, we observe that, MT achieves the highest data rate, but at the same time it drives the network-induced error into unacceptable region. Clearly, this is a consequence of unfairness, if an unstable sensor happens to be on the edge of the cell, it will never get an opportunity to send the measurements to the controller. To avoid visual clutter, Figs. 5b) and e) are showing a close-up on the performance of all schedulers without MT.

Additionally, we see that the optimal schedulers, MEF (opt) and MPEF (opt) perform very close to their greedy counterparts in terms of minimizing network-induced error. Interestingly, these control performance gains (if any) come at the cost of higher bandwidth utilization, especially for MEF: a decrease of the network-induced error by only $\approx 1\%$ results in 28 $\%$ higher bandwidth utilization (for $N = 95$). We expect the optimal schedulers to perform better than their greedy counterparts in a scenario with higher channel variations, hence more diverse resource demands.

We can observe that the proposed MPEF scheduler outperforms both RR and PF, and shows similar error-dependent dynamics as MEF. MPEF keeps the network-induced error low, while relying only on the static parameters of control systems. Figs. 5c) and f) confirm the performance trends for the network-induced error, expressed in the state error norm (for cart position and pendulum angle), and the number of nodes violating the requirements. State error is reduced up to 30 $\%$ by MPEF compared to closest performance by PF, and the number of requirements violations is reduced up to 2.5 times.

1) **MEF vs. MPEF trade-off:** As discussed in Sec. III-B1, implementation of MEF relies on the presence of an estimator at the sensor side. For most cases (low complexity and low cost sensors, distributed sensors, etc.), this implementation would not be possible [19], therefore, MPEF scheduling approach presents a viable alternative.

However, in the cases where MEF scheduling is implementable, it still requires communication overhead. Every sensor, prior to scheduling decision, needs to report (e.g., with control information elements) the current network-induced error to the BS. This implies a scheduling overhead per sensor $\delta_{SCH}^i$, thus more uplink resources are consumed. To evaluate the effect of the overhead, we define a total error cost function $C(N)$:

$$C(N) = \sum_{i=1}^{N} (1 + \delta_{SCH}^i) \frac{1}{T_D} \sum_{k=1}^{T_b} ||e_k^i||. \quad (18)$$

Intuitively, the term $\delta_{SCH}^i$ can be viewed as the amount of additional uplink resources for error reporting in the case of MEF. Hence, the function $C(N)$ shows the total network-induced error among all control sub-systems, with the additional term accounting for overhead. For MPEF scheduler $\delta_{SCH}^i = 0$, as it only relies on offline information. For simplicity of the evaluation, we further assume that the overhead is the same for all sensors, i.e., $\delta_{SCH}^i = \delta_{SCH} \forall i$. Fig. 6 presents the resulting dependency $C(N)$ for MPEF, and MEF for different overhead $\delta_{SCH}$. We see that in the $\delta_{SCH} = 0$ case (realistic for downlink only), MEF outperforms MPEF, while for the case of $\delta_{SCH} = 0.12$, their performances are almost identical. For all values $\delta_{SCH} > 0.12$, overhead becomes significant, therefore, MPEF performs close to MEF. If the number of states of the control system $n_i$, as defined in Sec. II is relatively low, or only some of the states are being measured, high overhead is very likely, and, hence, MPEF usage is favorable.

**V. Conclusions**

In this paper, we have considered the problem of allocating wireless resources in a cellular network for the networked control system of LTI independent control loops. We have devised a novel scheduling method (both greedy and optimal), based on the utility defined by the prediction of the networked induced error - Maximum Predicted Error First (MPEF) scheduler. We have shown via a comprehensive case study that the proposed MPEF demonstrates performance close to the optimal Maximum Error First (MEF) approach, but without the need for knowing the instantaneous error values of all control systems. Compared to MEF, proposed MPEF can be used in the case where no estimation at the sensor side is possible, and, for other cases, MPEF is outperforming MEF for the overhead more than 12 $\%$.

Future work in this area can address the stability of the control systems, and develop scheduling algorithms with stability bounds as a part of the utility function. Also, more sophisticated resource allocation models, including power constraints and variations of channel quality within a given bandwidth, can increase the practical value of the approach presented in our paper.
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REFERENCES


